Hong Kong Mathematics Olympiad (2018/19)
Heats (Individual)
香港數學競賽 (2018/19)
初賽項目(個人)

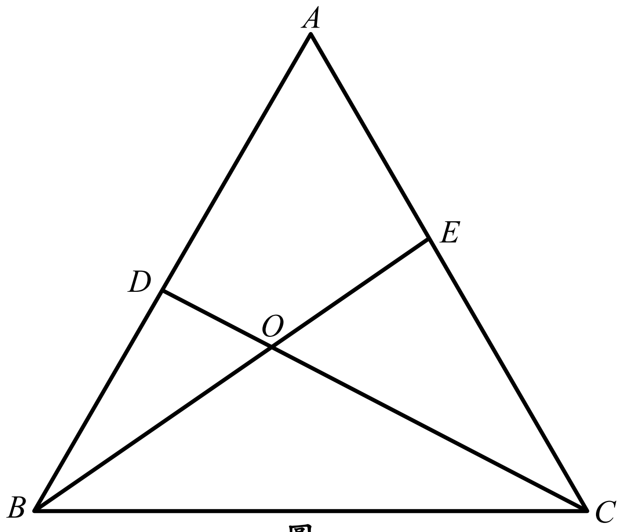
除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

甲部

Part A

1. 在圖一中，ABC 是一個等邊三角形。D 和 E 分別是 AB 和 AC 上的點使 $AE = BD$。若 CD 和 BE 相交於 O 及 $\angle COE = y°$，求 y 的值。

   In Figure 1, ABC is an equilateral triangle. D and E are points on AB and AC respectively such that $AE = BD$. If CD and BE intersect at O and $\angle COE = y°$, find the value of y.



圖一

Figure 1

2. 設 O 為極坐標系統的極點。若 $P(6, 240°)$ 向右平移 16 單位至 Q 而 ΔOPQ 的面積為 T 平方單位，求 T 的值。

   Let O be the pole of the polar coordinate system. If $P(6, 240°)$ is translated to the right by 16 units to Q and the area of ΔOPQ is T square units, find the value of T.

3. 己知 x 及 y 均為實數。若 $y^2 - 4xy + 5x^2 - 8x + 16 = 0$ 及 $F = x - y$，求 F 的值。

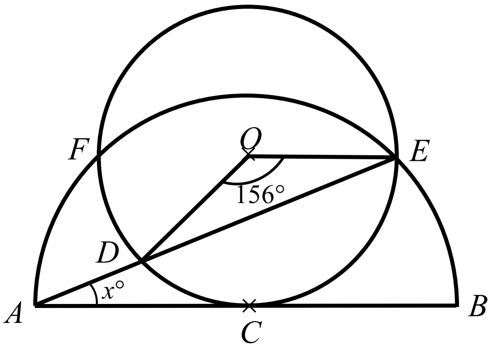
   Given that x and y are real numbers. If $y^2 - 4xy + 5x^2 - 8x + 16 = 0$ and $F = x - y$, find the value of F.

4. 設 n 為正整數。若 $a_n = 1+2+2^2+\cdots+2^n$ 及 $b = a_{10} - a_5 + a_1$，求 b 的值。

Let n be a positive integer. If $a_n = 1+2+2^2+\cdots+2^n$ and $b = a_{10} - a_5 + a_1$, find the value of b.

5. 在圖二中，O 為圓 DEF 的圓心。AB 為圓 DEF 在 C 的切線，其中 C 為半圓 ABE 的圓心，且半圓 ABE 通過 F。ADE 為一直線。若 $\angle DOE = 156°$ 及 $\angle BAE = x°$，求 x 的值。

In Figure 2, O is the centre of the circle DEF. AB is the tangent to the circle DEF at C, where C is the centre of the semicircle ABE, which also passes through F. ADE is a straight line. If $\angle DOE = 156°$ and $\angle BAE = x°$, find the value of x.

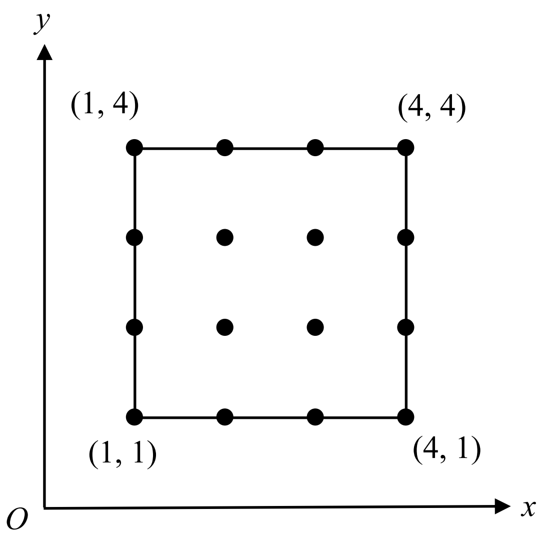


圖二

Figure 2

6. 在圖三中，直角坐標平面上一個正方形的四個頂點的坐標分別為(1, 1)、(1, 4)、(4, 1) 及 (4, 4)。若在該正方形中（包括邊界）選擇任何三個坐標均為整數的點，問可組成多少個 三角形?

In Figure 3, the vertices of a square in the rectangular coordinate plane are (1, 1), (1, 4), (4, 1) and (4, 4). How many triangles can be formed by selecting any three points in the square (including the boundary) with integer coordinates?

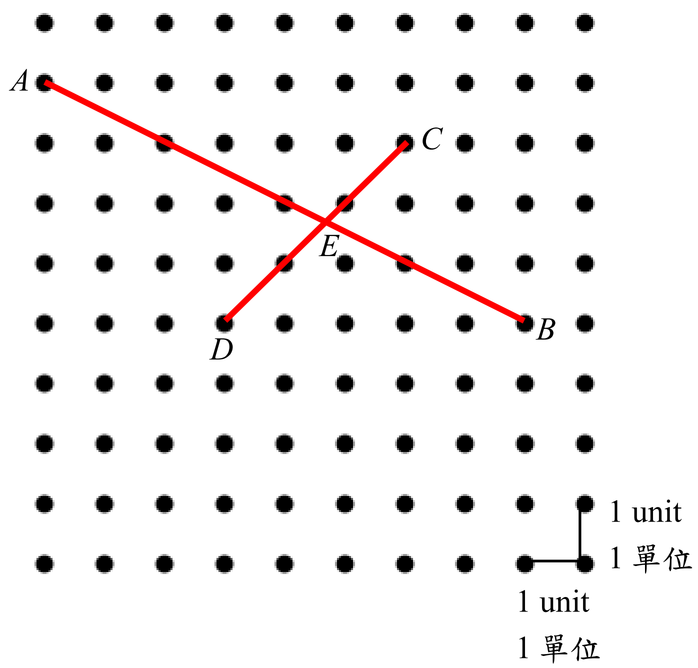


圖三

Figure 3

7. 在圖四中，AB 與 CD 相交於 E。設 q 單位為 AE 的長度。求 q 的值。

In Figure 4, AB and CD intersect at E. Let q units be the length of AE. Find the value of q.

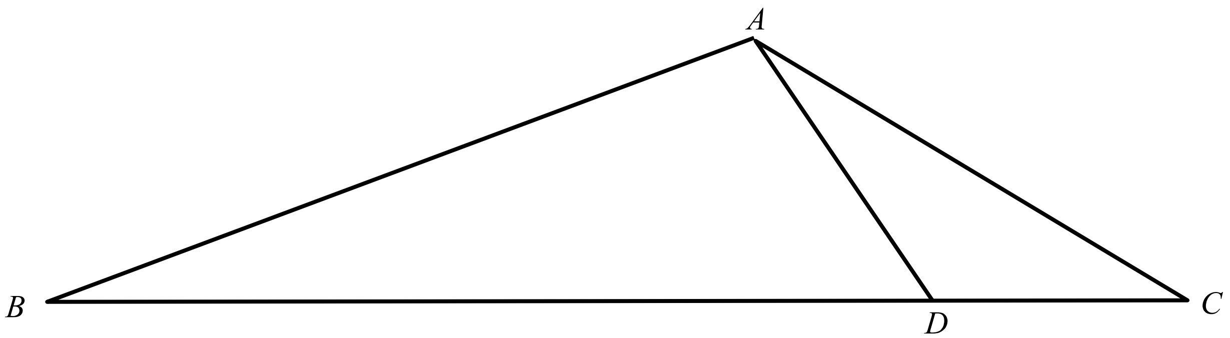


圖四

Figure 4

8. 在圖五中，D 是在 BC 上的一點使 $\angle ABD = \angle CAD$ 及 $\frac{BD}{AC} = \frac{8}{3}$。若 $\frac{\Delta ABD \text{ 的面積}}{\Delta ADC \text{ 的面積}} = k$，求 k 的值。

In Figure 5, D is a point on BC such that $\angle ABD = \angle CAD$ and $\frac{BD}{AC} = \frac{8}{3}$. If $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ADC} = k$, find the value of k.



圖五

Figure 5

9. 已知 α 及 β 為方程 $x^2+32x-1=0$ 的兩個根。若 $P=(\alpha^2+31\alpha-2)(\beta^2+33\beta)$，求 P 的值。

Given that α and β are the two roots of the equation $x^2+32x-1=0$. If $P=(\alpha^2+31\alpha-2)(\beta^2+33\beta)$, find the value of P.

10. 設 $c=\sqrt[3]{7+5\sqrt{2}}+\sqrt[3]{7-5\sqrt{2}}$。若 $w=c^2$，求 w。

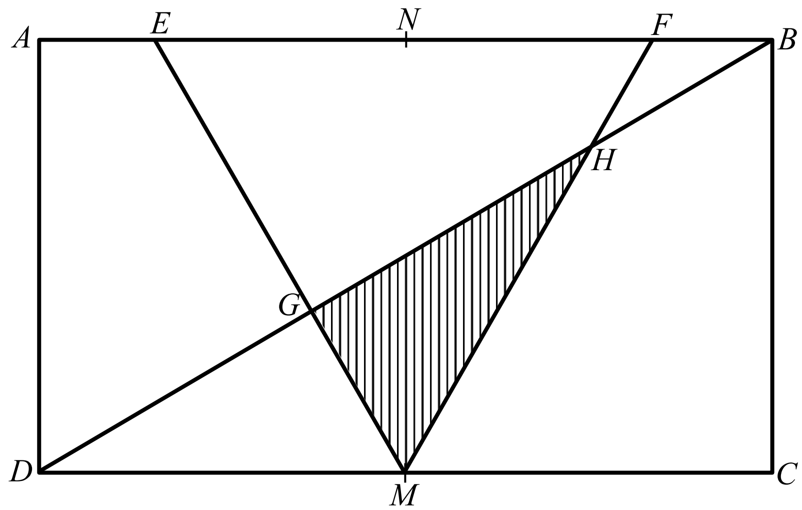
Let $c=\sqrt[3]{7+5\sqrt{2}}+\sqrt[3]{7-5\sqrt{2}}$. If $w=c^2$, find w.

乙部

Part B

11. 在圖六中，$ABCD$ 為一個長方形。M 和 N 分別是 DC 和 AB 的中點且 $AE:EN=BF:FN=1:2$。EM 和 FM 分別相交 DB 於 G 及 H。若長方形 $ABCD$ 及三角形 GHM 的面積分別是 96 和 S，求 S 的值。

In Figure 6, $ABCD$ is a rectangle. M and N are the mid-points of DC and AB respectively and $AE:EN=BF:FN=1:2$. EM and FM intersect DB at G and H respectively. If the areas of the rectangle $ABCD$ and the triangle GHM are 96 and S respectively, find the value of S.



圖六

Figure 6

12. 在三角形 ABC 中，$AB=14$、$BC=48$ 及 $AC=50$。將 P 及 Q 分別記為ΔABC 的內心及外心。設 d 單位為 PQ 的長度。求 d 的值。

In triangle ABC, $AB=14$, $BC=48$ and $AC=50$. Denote the in-centre and circumcentre of ΔABC by P and Q respectively. Let d units be the length of PQ. Find the value of d.

13. 已知正整數 a、 b 及 c 滿足下列條件：

(i) $a > b > c$，

(ii) $(a-b)(b-c)(a-c) = 84$，

(iii) $abc < 100$。

設 M 為 a 的最大值。求 M。

Given that a , b and c are positive integers satisfying the following conditions:

(i) $a > b > c$,

(ii) $(a-b)(b-c)(a-c) = 84$,

(iii) $abc < 100$.

Let M be the maximum value of a . Find M .

14. 已知 $3\sin x + 2\sin y = 4$ 。設 N 為 $3\cos x + 2\cos y$ 的最大值。 求 N。

Given that $3\sin x + 2\sin y = 4$. Let N be the maximum value of $3\cos x + 2\cos y$. Find N .

15. 已知 x、y、 z 為正實數且滿足

$$\begin{cases} x^2 + xy + y^2 = 7 \\ y^2 + yz + z^2 = 21 \\ x^2 + xz + z^2 = 28 \end{cases}$$

若 $a = x + y + z$， 求 a 的值。

Given that x, y, z are positive real numbers satisfying

$$\begin{cases} x^2 + xy + y^2 = 7 \\ y^2 + yz + z^2 = 21 \\ x^2 + xz + z^2 = 28 \end{cases}$$

If $a = x + y + z$, find the value of a.

完

END